The wave drag of wind over water

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It is shown to be probable that a large proportion of the drag exerted by a water surface on the wind is in the form of wave drag. As a result the usual relation between the wind profile and the surface stress and roughness length are modified. In particular, close to the surface the relation between the transport of momentum and that of heat and water vapour are different from that obtaining over a rough solid surface.

1. Introduction

The interaction between wind and a water surface seems to have been considered from two points of view. In one, favoured by meteorologists, the water surface is regarded as a boundary of the same general nature as solid ground, with negligible velocity relative to that of the wind, which exerts a shearing stress upon the air. From this point of view the principal problem is to determine the characteristic 'roughness length' of the water surface (e.g. see Ellison 1956).

On the other hand there is a growing body of literature (Eckart 1953; Ursell 1956; Phillips, 1957, 1958; Miles 1957, 1959, 1960; Shuleykin 1959) in which the problem of wave generation by wind is treated, the nature of the air flow being taken as given.

An important aspect of this problem seems to have been overlooked. It will be shown below that of the momentum withdrawn from the air by interaction with the water surface, a far from negligible proportion goes into *wave* momentum. It is not difficult to show that the effective height at which the air loses this momentum is not the surface, but is in fact some height at which the wind speed is not less than the phase speed of the waves. We therefore conclude that over water the lowest levels of the atmosphere cannot be considered to be a region of constant turbulent stress, as is usually assumed, but that the turbulent stress must increase with height above the surface. We should thus expect modification of the wind-velocity profile over water and failure of the usual relation between this profile and the surface stress.

2. Momentum input to waves

Although there remains considerable uncertainty about the exact nature of the waves produced by a wind of given strength, duration and fetch, the body of empirical data is now so large that no modern compilation can be very far from the truth. The most convenient presentation is probably that of Groen &

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Dorrestein (1958). They have used virtually all the data on wind waves published prior to 1957 to obtain average wave characteristics as a function of wind duration and wind speed. In their Diagram I, wave height H and wave period T is plotted against duration t with wind speed U as a parameter. I have taken from these curves the values for U, t, T and H given in table 1. The curves can be read reliably only to two significant figures, but this accuracy is at least as good as the data upon which they are based.

U (m/sec)	t (hr)	T (sec)	H (m)	E (j/m^2)	$C = gT/2\pi$ (m/sec)	$E/C = M$ (kg sec^{-1} m^{-1})	M/t (kg sec^{-2} m^{-1})
10	0·5	1·9	0·32	126	2·96	43	$2{\cdot}4 \times 10^{-2}$
	1·0	2·5	0·53	340	3·9	87	2·4
	2·0	3·2	0·88	950	5·6	190	2·6
	3·0	3·8	1·14	1,590	5·9	270	2·5
	6·0	4·7	1·55	2,940	7·3	462	1·9
15	0·5	2·5	0·54	360	3·9	92	5·1
	1·0	3·2	0·89	970	5·0	194	5·4
	2·0	4·1	1·45	2,570	6·4	40	5·6
	3·0	4·7	1·95	4,650	7·3	64	5·9
	6·0	6·2	2·95	10,600	9·7	109	5·1
	9·0	7·1	3·5	15,000	11·1	135	4·2
20	0·5	2·9	0·77	720	4·5	16	8·9
	1·0	3·8	1·26	1,940	5·9	33	9·2
	2·0	4·9	2·15	5,700	7·65	745	10·4
	3·0	5·8	2·85	9,900	9·05	109	10·0
	6·0	7·5	4·5	24,800	11·7	212	9·8
	9·0	8·6	5·6	38,400	13·4	287	8·9
	12·0	9·4	6·3	48,600	14·7	338	7·8

Table 1

Wind-driven waves are certainly not completely irrotational. On the other hand the relations between wavelength, wave period and phase speed observed for such waves do not differ greatly from those of irrotational waves. It seems unlikely that the momentum associated with the wave motion is greatly different from that of an irrotational wave of the same height and period. If anything, the momentum in the real wave may be somewhat greater than that in the irrotational one.

It is not difficult to show (Lamb 1932, §250) that in an irrotational wave the relation between the wave momentum per unit area M and the wave energy per unit area E is

$$M = \frac{E}{C} = \frac{2\pi}{g} \frac{E}{T}. \tag{1}$$

In an actual sea a fairly wide spectral band of wavelengths contributes to the energy, but it is difficult to see how the relations between momentum, energy density and a characteristic wave period could differ greatly from (1).

With these assumptions I have listed in table 1 the wave momentum corresponding to the data shown from Groen & Dorrestein. The average rate of increase of wave momentum can be obtained by dividing by the wind duration t.

An inspection of the last column of table 1 shows that the average rate of momentum increase, M/t, is nearly constant for a very considerable time after the onset of the wind. At later times, of course, saturation effects dominate, and the momentum input must be balanced by losses due to wave breaking, turbulence and viscosity.

The data of table 1, for sufficiently small t, may be summarized by

$$\frac{M}{tU^2} = 2{\cdot}4 \times 10^4 \quad \mathrm{kg\,m^{-3}}. \tag{2}$$

Now $M/tU^2\rho$, where ρ is density of air, has the nature of a drag coefficient, say

$$C_{dW} = 2 \times 10^{-4}, \tag{3}$$

if we define the total drag coefficient C_d by putting the stress

$$\tau = C_d \rho U^2. \tag{4}$$

The total drag coefficient given by Ellison (1956) for winds of about 15 m/sec, using (4) as a definition of C_d, varies only slightly with U and has an average value

$$C_d = 10 \times 10^{-4}. \tag{5}$$

Clearly, then, the contribution of the wave drag (3) to the total drag (5) is far from negligible. Moreover, the estimate of the wave drag given in (3) is really only a lower limit. All the momentum which was originally put into waves, but which has been lost to the drift current because of wave-dissipation mechanisms, is omitted from the estimate. It will be seen below that there are reasons for believing that a majority of the momentum is transferred by wave drag.

3. Interaction height

The energy to momentum ratio of a surface wave is equal to the phase speed C. If an interaction with the air transfers energy and momentum to the waves, then for a given quantity of momentum transferred the energy lost by the air must be at least equal to the energy gained by the waves.

Thus,

$$d(1/2\rho U_i^2) \geqslant C d(\rho U_i), \tag{6}$$

or

$$U_i \geqslant C,$$

where U_i is the wind velocity at the height at which the reaction of the air to the wave drag is effective. Miles (1960) finds that the rate of increase of wave energy depends upon the variation of the velocity profile at the point where $U = C$. This is consistent with (6), as is the reasonance mechanism proposed by Phillips (1957).

4. Energy and momentum transfer

A brief examination of the means by which energy and momentum are transferred from the air to the wave is warranted. In an ordinary turbulent shear flow over a solid surface, momentum is transferred towards the boundary by a Reynolds stress τ. If the boundary is smooth, viscous shear stress ultimately takes over, while if it is rough the force on the boundary is exerted by pressures on the roughness elements. Energy passes towards the boundary by the action of the Reynolds stress in a term of the form τU. There is also a transfer due to the covariance $-\overline{pv}$ of the pressure fluctuation p and the vertical turbulent velocity $-v$, as well as the 'turbulent energy diffusion' $-\frac{1}{2}\rho\overline{q^2v}$, where q is the local instantaneous speed of the fluctuating motion (Townsend 1955). If the boundary is stationary, no energy goes into it, all the energy being dissipated by the turbulence and the viscous sublayer.

In the case of wave generation the situation is somewhat different. The momentum will be carried right to the surface by the Reynolds stress, as in rough flow. Energy is again transferred to the extent $\tau U - (\overline{pv} + \frac{1}{2}\rho\overline{q^2v})$. However, the energy going into the waves must pass unattenuated through the region where $U < C$. Thus, as τU decreases, $-(\overline{pv} + \frac{1}{2}\rho\overline{q^2v})$ must increase to compensate.

Motions of the required nature must be too highly organized to be called turbulence, particularly since no absorption of energy is involved. Highly organized motions of the required type are involved in Miles's (1960) discussion. They are not described, but must also occur, in the mechanism described by Phillips (1957). In Miles's case the motions are in response to the moving surface, and a Reynolds stress is developed which remains constant with increasing height to the level where $U = C$ and then drops to zero. In Phillips's case the motions must be in response to moving pressure fluctuations in the air at levels $U \geqslant C$, and to movements of the surface caused by these pressures.

5. 'Rough' air flow and wave generation

It is generally believed (e.g. see Ursell 1956, Ellison 1956) that the water surface is at least nearly aerodynamically 'rough' so far as its influence on the motion of the air is concerned. To the extent that this belief is justified, then, the momentum of the air is transmitted into the water because of the correlation between local pressure and local surface slope. Looked at from the point of view of the water, we find that the water is receiving momentum from the air by pressure forces.

Now, in a homogeneous fluid pressure can only produce irrotational motion, and it is difficult to conceive of an irrotational motion carrying momentum which does not have the character of a surface wave, provided water far below the surface is stationary. (The only motion meeting the requirement in the case of a large surface of water seems to be one in which there is a correlation between particle velocity and surface elevation, and it is difficult to see how such a motion would differ from that of a wave.) This argument leads to belief that perhaps most of the momentum from the air enters the water in the form of wave motion. Of course much may go into very small waves or ripples which rapidly decay and lose their momentum to the drift current. The data of Groen & Dorrestein are certainly not inconsistent with this view.

6. Consequences

We have seen above that momentum and energy going into waves of phase velocity C must pass unattenuated through the portion of the air column for which $U < C$. As was remarked, the motions which carry this stress and energy cannot be described as turbulence. Above the level where $U = C$, however, this stress will be carried by the turbulence. We therefore conclude that the stress carried by the turbulence must *increase* with height above the water surface.

Any discussion of the relation between the turbulent intensity and the mean velocity gradient, whether or not employing the concept of 'eddy viscosity', must lead to the conclusion that the near-surface velocity shear is less in the present case than over a solid boundary. The simple dimensional argument:

$$\frac{\tau_t}{\rho z^2} \propto \left(\frac{\partial U}{\partial z}\right)^2, \tag{7}$$

yields the result that the velocity gradient will be reduced as the square root of the turbulent stress. Here τ_t has been written for the turbulent shear stress.

The usual method of analysis is to assume that the mean velocity will have the form

$$U = \frac{u^*}{K} \ln\left(\frac{z}{z_0}\right), \tag{8}$$

where $u^* = (\tau/\rho)^{\frac{1}{2}}$ and k is von Karman's constant $= 0{\cdot}4$. Measurements of U are made at a variety of heights, so that τ and the roughness length z_0 may be determined.

The consequence of the increase of turbulent stress with height may be expected to be a reduction in $\partial U/\partial z$ at low levels compared with that occurring over a solid surface with the same total stress. The usual interpretation of the data is likely to yield too low a value for the shear stress, and a value for the roughness length which will increase as the height of observation increases.

The most important consequence, however, is likely to be in making estimates of the turbulent transports of heat and water vapour. At the height at which most observations are taken, it is probable that most of the stress is carried by the turbulence, and the error in the calculated total stress may be small, although the significance of the calculated z_0 may be doubtful. However, if the usual interpretation is put upon the data there there will be a large overestimate of the intensity of the turbulence close to the surface. The motions transferring momentum and energy to the water waves will be organized and wave-like, and presumably will contribute little to the transport of sensible heat and water vapour. Thus, in the critical region close to the surface, the transport coefficients for heat and water vapour may be much lower than might be inferred from the momentum transport. It is quite likely, then, that the temperature gradients and humidity gradients close to the water surface will be much larger than would be anticipated from the usual assumptions. It would be useful to have some experimental results to check this conclusion.

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